

SORTING TECHNIQUE SIMPLIFIED

MERGING  
SORTING

HAL H DAVIES AUGUST 1 1963

ABSTRACT

SORTING TECHNIQUES SIMPLIFIED

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July 15, 1963

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"Sorting Techniques Simplified" is an analysis of internal sorting and merging techniques used in computer sorting. Simplified examples are used with each method of sorting covered. There is no attempt to evaluate the technique as to efficiency or speed, only in explaining how the technique works. The techniques are divided into two major categories:

1. Internal Sorting
2. Merging

1. General  
2. Introduction  
3. The Home

4. Introduction

(1) General Working

(2) Change Switching

(a) Single Exchange

Example 11 - Single Exchange

(b) Interchanging Channels

Example 12 - Interchanging Channels

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## Section I

### INTRODUCTION

Much of the literature about sorting deals either with a specific sort or in the evaluation of a particular sorting technique. In this paper, the mathematical evaluation is intentionally missing in order to facilitate the understanding of 'how it works'. There are, however, some generalized statements about various methods to help place their usage in a program or in a generalized sort.

An effort has also been made to make examples independent of a machine or a device. In some of the many examples pages are shown as input/output devices. This is not to exclude usage of any other device.

The Glossary of Terms is a collection of terms found in IBM and other sorting materials to help the inexperienced understand the particular technique or evaluation being studied. They are not necessarily new, in this paper.

Generalized sorts can be arbitrarily divided into three sections:

#### A. Assignment Phase

The function of this phase is to establish the parameters for the particular sort. Read and evaluate control cards, determine blocking factors and record length, and calculate "C". The control fields (h.o.s.) are determined and program modifications made to compare them. The assignment phase turns a generalized sort package into a specific sort.

#### B. Interval Sort Phase

Interval sort develops string of records. "C" is a sort used to describe the array size in number of records that can be accommodated in the interval sorting phase of the program. It is equally desirable to have the output, consecutive strings, from this phase to be as long as possible to minimize the time necessary in the next phase. In the examples of techniques used in interval sorting all numbers used are four (control fields). They are limited to two digit numeric values to facilitate independence of the techniques. In actual usage most generalized sorts will handle alphabetic or special characters in the keys. All are assumed to be on ICDS that will only compare one at a time whether locked or unlocked to the sort process. In some cases a "lock" may be assumed (not in sequence) at the technique start.

### B. Internal Sort Phase (continued)

The number series used is continued through all examples. It is thought to be a random series, but this is not a necessity. It is not used to show the best or worst of the techniques, only to serve as an example. Keeping the same series may help the reader learn and understand the method.

### C. Merging Phase

Merges start with in-sequence strings of records developed in the internal sort phase. In this phase the strings are brought together in each pass developing longer and longer strings until in the final merge pass (sometimes called a separate phase) records are merged into one string, which is the output of the sort program.

Notions have been placed in their relative position to facilitate understanding, i.e. some techniques use a combination of other techniques or area modification of another technique. These will follow in sequence. The sequence is not to indicate merit or extent of range of a method.

For this paper many ideas and examples have come from the papers presented at the Association of Computing Machinery Sort Symposium, December 1962 and recorded in Communications of the Association of Computing Machinery, Volume 6, Number 5, May 1963. A special note of gratitude goes to G. A. Gottlieb of the University of Toronto, Canada for his paper "Sorting on Computers".

It is hoped that this paper can help the reader in a basic understanding of the techniques used in sorting.

## Section II

### INTERNAL SORTING

In writing of a sort there are many factors that determine techniques to use. Some of these factors are:

1. Number of compares needed to place string in sequence.
2. Available memory size (program steps needed as opposed to size of G). The larger the G the less time is needed to merge.
3. Balance of Input Output speed to processing speed.
4. Features of the computer used.

The decision of what technique to use in a sort comes down to speed (running time considerations). It is desirable to maximize the length of the original strings and minimize the number of compares, but keep the processing time close to equal to the Input Output speed of the computer. A good method in one situation may be a poor method in another. Generalized sorts must also be able to handle a wide variety of situations. Specific sorts can be tailored to a situation. Hence no single method could be singled out as best for all.

#### A. Exchange Techniques

Exchange starts with a full "G" and proceeds to exchange records until the desired sequence is achieved.

##### (1) Simple Exchange (Bisection)

Method:

Each key is compared with the following key. If the first is greater, the two are exchanged. If the first is smaller, the compare continues 2nd with 3rd etc. In the case of a switch no attempt is made to use the higher of the two in the next compare. The next compare is with the following two records. If there is a switch between Record 1 and Record 2, Record 3 and Record 4 constitute the next compare. This process is continued until the end of the G area is reached. Each scan of the entire G area constitutes a pass. The passes are continued until no exchanges are made on a pass. The records are then in sequence.

## Bubble Exchange (continued)

longer:

This method is used in scientific type program where a specific group of fields are sequenced by a Fortran "DO" loop. The number of instructions generated is small, but the number executed is large. This method is highly dependent upon distribution of the keys. In the case where the low key is last in the file or a reverse sequence, it requires "N" passes to order file. If the file contains just a few keys reversed, 1 or 2 passes will be sufficient.

Example #1:

The 19 - 21, 22 - 27, and 31 - 38 are exchanged in the first pass. All other pairs are in sequence. In the 2nd pass only the 22 - 21 are exchanged. A 3rd pass not shown would be necessary to determine that records were in sequence.



Example #1

SINGLE SWAPING (BY SECTION)

Original Seq. #	Seq	Seq	Seq	Passes			Seq	Seq
				1st	2nd	3rd		
00	01	07	07	01	02	02	01	01
01	10	10	10	10	10	10	10	10
02	20	25	25	20	21	21	21	20
03	05	05	05	02	26	11	01	01
04	07	07	01	03	31	30	20	20
05	00	21	27	11	00	30	30	30
06	02	02	13	07	30	42	41	03
07	01	11	32	13	07	55	55	35
08	30	50	30	02	05	07	07	07
09	07	55	55	50	00	92	10	07

## 10) Selection Exchange

### Method:

The technique here is very similar to the simple exchange. An attempt is made to move high keys to the end and the low keys to the start more quickly. In the first pass the compare starts with the first two records and compares follow as long as records are in sequence. The difference occurs on an exchange. The high of an exchange is compared with the next record. This comparing continues until a higher record at the end of it is found. In the meantime all records passed are shifted up one position. A key can only move up one position but down many. The second pass functions in reverse placing the smaller keys closer to the start of the "G". A key exchanged can move down one but up many. The following passes alternate, odd number pass ascending, even number passes descending.

### Use:

More program steps are required but a small number of compares and exchanges is needed than in simple exchange. Consequently, fewer passes on a reverse sequence file S-I pass are necessary. Probably the best usage would be in sorts that are necessary within a program rather than in a generalized sort.

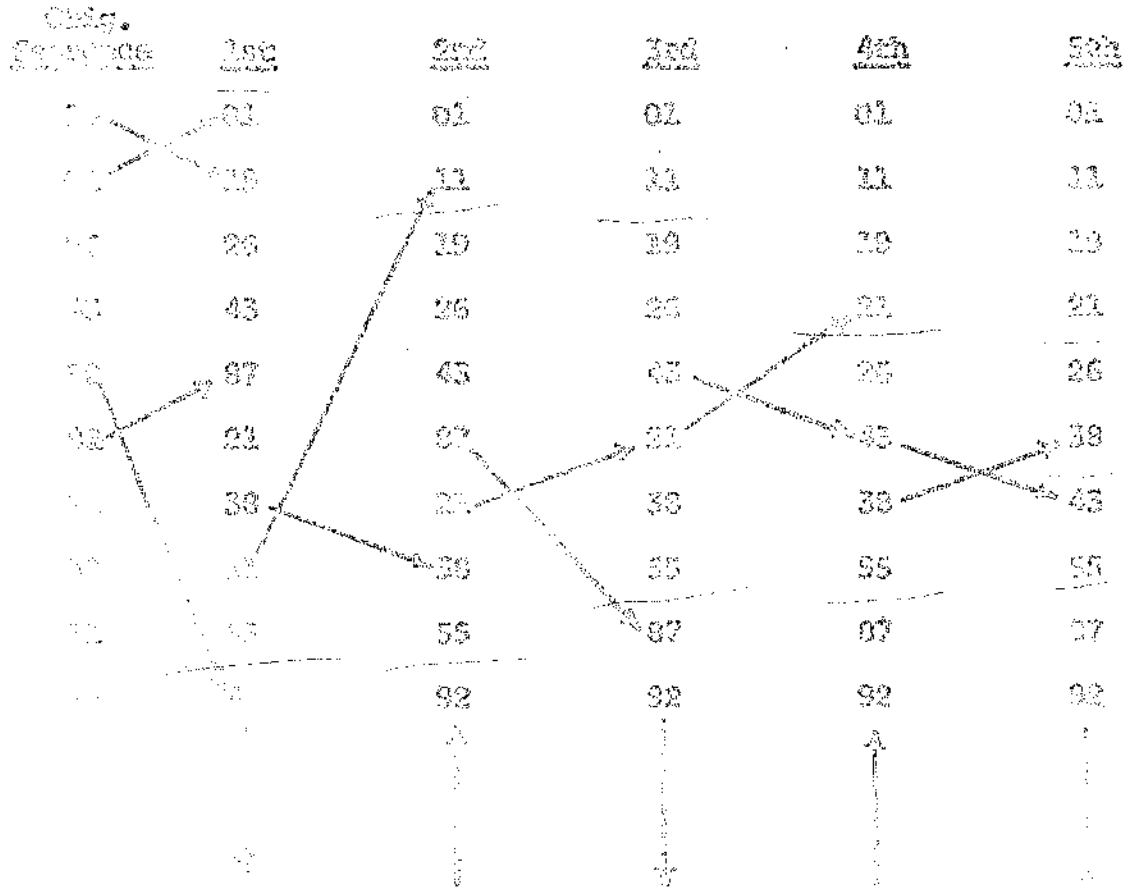
### Example 11:

In the first pass proceeding downward, recognizes that the 10 and 21 are out of sequence, exchanges them and compares the 10 to the 25. The 10 is 10000, a further shift is executed. Comparing continues until the 20 and 21 are compared. In this out of sequence the 21 is shifted to 1. The 20 is now the next compare and is higher than 11, 22, 11 and 25. The 20 is then placed in the last 0 position with the other records, shifted up one position each.

In the second pass the compares start with the 22 and function in the opposite direction. The first out of sequence are the 11 and 20. The 20 drops down and the 11 moves up to the second position. A sixth pass not shown is needed to check for completion of sequencing.

Example 10

PERMUTATION EXCHANGE



Arrows indicate direction of pass.

Arrows by numbers show movement of keys.

## Binary Radix Exchange

### Method:

Scan is started on the beginning of "0", binary bit position of key. The first one bit encountered is exchanged with the first zero bit encountered starting the scan from end of "0". The scan is continued and exchanges are made until the beginning scan meets the end scan. An indicator is then placed at this position to establish the definition of the same for following passes. Above the indicator all positions are "0", below all are "1". The next high order position is used as an indicator, set to the previous pass indicating the number of scans on the pass.

The process continues until the entire bit positions have been scanned. Within point all records are in sequence. Each record which contains will have an indicator set.

### Uses:

Binary radix exchange is particularly adapted to the binary computers. The length of the key can be the limiting factor in that the passes are equal to the number of bits in the record field.

### Example:

In the above case all numbers less than 64 fill the first group. 64 and 99 fill the second. The range continues to decrease by the power of 2. The indicators are placed at 64 on first pass, 32 and 96 second, 16, 48 etc. on third, until the last pass where there is an indicator placed on all groups except equals. (The brackets show an exchange (always first with last). The V indicates bit position scanned. The lines show definition areas.

Example 21

SAINT-EXUPÉRIE (1910-1922)

I	II	III	IV
19	000000	19	000001
20	000001	20	000002
21	000002	21	000003
22	000003	22	000004
23	000004	23	000005
24	000005	24	000006
25	000006	25	000007
26	000007	26	000008
27	000008	27	000009
28	000009	28	000010
29	000010	29	000011
30	000011	30	000012
31	000012	31	000013
32	000013	32	000014
33	000014	33	000015
34	000015	34	000016
35	000016	35	000017
36	000017	36	000018
37	000018	37	000019
38	000019	38	000020
39	000020	39	000021
40	000021	40	000022
41	000022	41	000023
42	000023	42	000024
43	000024	43	000025
44	000025	44	000026
45	000026	45	000027
46	000027	46	000028
47	000028	47	000029
48	000029	48	000030
49	000030	49	000031
50	000031	50	000032
51	000032	51	000033
52	000033	52	000034
53	000034	53	000035
54	000035	54	000036
55	000036	55	000037
56	000037	56	000038
57	000038	57	000039
58	000039	58	000040
59	000040	59	000041
60	000041	60	000042
61	000042	61	000043
62	000043	62	000044
63	000044	63	000045
64	000045	64	000046
65	000046	65	000047
66	000047	66	000048
67	000048	67	000049
68	000049	68	000050
69	000050	69	000051
70	000051	70	000052
71	000052	71	000053
72	000053	72	000054
73	000054	73	000055
74	000055	74	000056
75	000056	75	000057
76	000057	76	000058
77	000058	77	000059
78	000059	78	000060
79	000060	79	000061
80	000061	80	000062
81	000062	81	000063
82	000063	82	000064
83	000064	83	000065
84	000065	84	000066
85	000066	85	000067
86	000067	86	000068
87	000068	87	000069
88	000069	88	000070
89	000070	89	000071
90	000071	90	000072
91	000072	91	000073
92	000073	92	000074
93	000074	93	000075
94	000075	94	000076
95	000076	95	000077
96	000077	96	000078
97	000078	97	000079
98	000079	98	000080
99	000080	99	000081

V	VI	VII	VIII
01	000001	01	000001
02	000002	02	000002
03	000003	03	000003
04	000004	04	000004
05	000005	05	000005
06	000006	06	000006
07	000007	07	000007
08	000008	08	000008
09	000009	09	000009
10	000010	10	000010
11	000011	11	000011
12	000012	12	000012
13	000013	13	000013
14	000014	14	000014
15	000015	15	000015
16	000016	16	000016
17	000017	17	000017
18	000018	18	000018
19	000019	19	000019
20	000020	20	000020
21	000021	21	000021
22	000022	22	000022
23	000023	23	000023
24	000024	24	000024
25	000025	25	000025
26	000026	26	000026
27	000027	27	000027
28	000028	28	000028
29	000029	29	000029
30	000030	30	000030
31	000031	31	000031
32	000032	32	000032
33	000033	33	000033
34	000034	34	000034
35	000035	35	000035
36	000036	36	000036
37	000037	37	000037
38	000038	38	000038
39	000039	39	000039
40	000040	40	000040
41	000041	41	000041
42	000042	42	000042
43	000043	43	000043
44	000044	44	000044
45	000045	45	000045
46	000046	46	000046
47	000047	47	000047
48	000048	48	000048
49	000049	49	000049
50	000050	50	000050
51	000051	51	000051
52	000052	52	000052
53	000053	53	000053
54	000054	54	000054
55	000055	55	000055
56	000056	56	000056
57	000057	57	000057
58	000058	58	000058
59	000059	59	000059
60	000060	60	000060
61	000061	61	000061
62	000062	62	000062
63	000063	63	000063
64	000064	64	000064
65	000065	65	000065
66	000066	66	000066
67	000067	67	000067
68	000068	68	000068
69	000069	69	000069
70	000070	70	000070
71	000071	71	000071
72	000072	72	000072
73	000073	73	000073
74	000074	74	000074
75	000075	75	000075
76	000076	76	000076
77	000077	77	000077
78	000078	78	000078
79	000079	79	000079
80	000080	80	000080
81	000081	81	000081
82	000082	82	000082

## (4) Merge Exchange

### Method:

This technique can be used to quickly bring low keys to the front of "G" and high keys to the end. For best results "G" should be a power of 2. The first pair compares the first record with first record in the second half of "G". If in sequence they are unchanged; if out of sequence, they are exchanged. This is continued with the second record in the first half with the second record in second half. This proceeds until the last record in the first half is compared with the last in the second half.

The next pair uses a 6 quartered i.e., third record in first quarter, 4th in second quarter, 5th in third quarter, 6th in fourth quarter. These records are placed in sequence. It may take more than one exchange to complete the sequencing. The process is continued until a section contains one record.

<u>Pass</u>	<u>Exchanges</u>
1	5/2
2	6/4
3	8/8
4	8/16
5	-
6	-
7	8/16

### Use:

This technique tends to decrease the number of compares to sort a "G". However, the number of exchanges tends higher than some other methods. Heavy use of balancing passes necessary to sort this approach practical. Possible improvement is shown in the next method. There is a tendency to have multiple exchanges at the final pass.

### Example #4:

In the first pass with a G equal to 16, the 29 is compared with the 11 and exchanged. The 91 is compared with the 55. No exchange occurs. Exchanges are made with the 22. In the third pass a double exchange occurs. The 34 is compared to the 26 and exchanged. 54 is then compared with the 63 no exchange. The 92 is compared with the 10 and exchanged. At this point sequence is still not correct. The 64 and 65 will be exchanged. Any multiple exchanges occur at the final pass.

Appendix II  
Inventory of

Category	Code	Inventory	Quantity	Category	Code	Inventory	Quantity
General	01	01	01	General	01	01	01
	02	02	02		02	02	02
	03	03	03		03	03	03
	04	04	04		04	04	04
	05	05	05		05	05	05
	06	06	06		06	06	06
	07	07	07		07	07	07
	08	08	08		08	08	08
	09	09	09		09	09	09
	10	10	10		10	10	10
Special	11	11	11	Special	11	11	11
	12	12	12		12	12	12
	13	13	13		13	13	13
	14	14	14		14	14	14
	15	15	15		15	15	15
	16	16	16		16	16	16
	17	17	17		17	17	17
	18	18	18		18	18	18
	19	19	19		19	19	19
	20	20	20		20	20	20

15. 11/1

16. 11/1

17. is basically the same exchange routine with the factors  
reversed. The formula used for the factor is  $2(C^2/4) + 1$ . The  
number portion only is used.  $C^2$  is divided by 4 to find the  
new factor until factor is equal to 1. i.e.:

- 5 = 11    factors are 5, 2, 1
- 4 = 16    factors are 4, 2, 1, 1
- 3 = 47    factors are 23, 2, 1, 1

18. 11/1

19. In these factors, less complex and less exchanges are needed  
to complete the case. Banks and factors (20th January 1960) find  
this change is effective with a "6" up to 255.

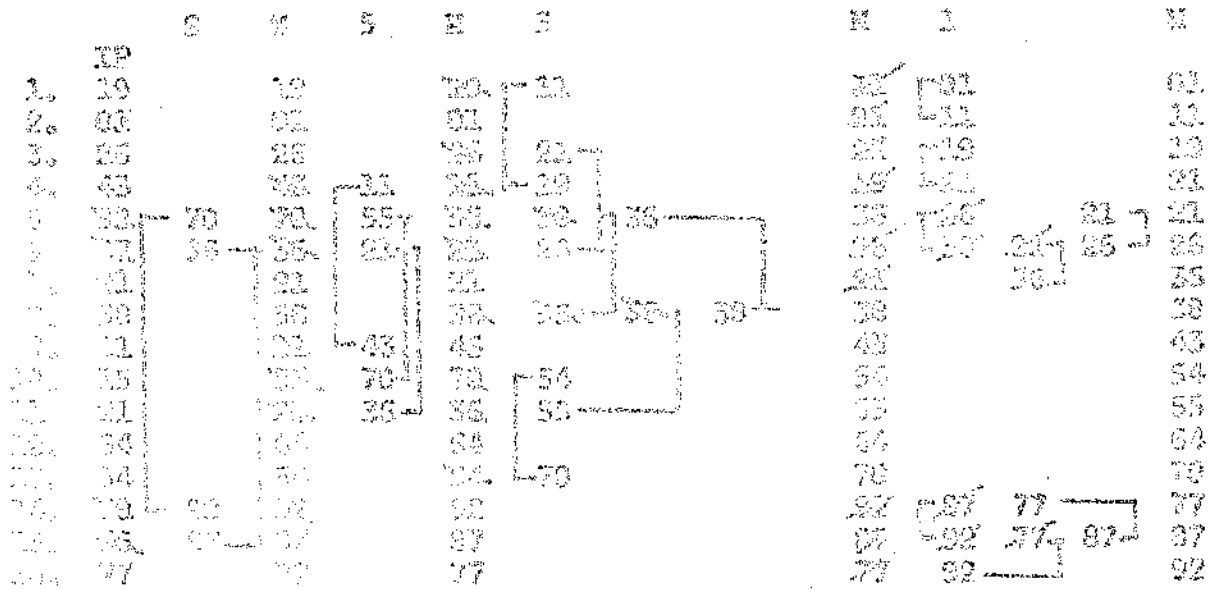
20. 11/1

21. Factors: page 1 - 2, page 2 - 3, page 3 - 4, page 4 - 5. The 19  
is compared with 55. The value change occurs at 19 compared with  
10. In this example the effect of adding an additional case with  
a factor of 2 is shown. The effect in this case is to double the  
number of exchanges and see our case sum of 6. Using the factor  
"2" has not as this has been seen. This case and some more  
needed to indicate some values may be shown.



Example 66

3187



After adding a factor 2 between 3 and 1

SM 2	S 1	Final M
11	11 01	01
01	01 11	11
21	01 21	21
19	21 11	11
30	21 30	30
25	30 21	21
37	30 37	37
38	37 30	30
36	37 36	36
34	36 37	37
35	36 34	34
34	35 34	34
33	34 35	35
32	33 34	34
31	32 33	33
30	31 32	32
29	30 31	31
28	29 30	30
27	28 29	29
26	27 28	28
25	26 27	27
24	25 26	26
23	24 25	25
22	23 24	24
21	22 23	23
20	21 22	22
19	20 21	21
18	19 20	20
17	18 19	19
16	17 18	18
15	16 17	17
14	15 16	16
13	14 15	15
12	13 14	14
11	12 13	13
10	11 12	12
9	10 11	11
8	9 10	10
7	8 9	9
6	7 8	8
5	6 7	7
4	5 6	6
3	4 5	5
2	3 4	4
1	2 3	3

## B. Input Machine

This method involves each digit of a key to determine sort. Binary code evidence is divided into exchange records and records too exchanged. The card number is the next example of such sorting.

### (1) Position Identification from the Card Content (Initial)

Method:

The key is obtained starting with the white position in the first pass and the record placed in the designated pocket or reserved memory area determined by the value of this key position. Records are then collected from the pockets keeping records in same order as they entered the pocket.

The next pass records the next digit position to the left, again placing records in designated pocket. Another collection is made. This procedure would be high order digit has been examined and the collection made. Records are now in key sequence.

Example:

The computer difficulties in this method center around two elements.

The variable pocket size and (2) the length of the key. However, this might be used in modified form to obtain sort using a high order digit as facilitator use of another technique. The result in each pass is then sorted in priority of the rest.

Example (1):

The key (10) white digit (0) is examined and placed in the 10<sup>th</sup> pocket. The key (00) is examined and placed in the 1<sup>st</sup> pocket. This is done for all records. Finally the (11) is examined and placed in the 1<sup>st</sup> pocket. The records are then collected using the first in first out method. In the first collection the (00) is taken from pocket #1 about, then (11), (10) and finally the record (01).

Each sorting is pre-counting on a high order digit to decrease the list to be sorted.

Example #6  
 KADER OR DISCREETION  
 (FOR THE 2000)

Example	Pocket #
70	0
81	
76	21, 11
48	21, 61
82	32
89	
11	28
31	
12	33, 34
27	
33	35
66	
34	36, 38
71	
72	37, 38
73	
74	39
75	

1000 2000 United  
 Number of Key

Example	Pocket #	
70	01	01
81		11
21	19, 11	19
11		21
81	25, 21, 21	21
82		25
48	38, 38	38
68		48
84	48	54
85		64
26	55, 54	55
36		64
87	64	76
77		77
80	77, 77	87
83		92
1st collection:	87	3 2nd col.
	88	9

1000 2000 Penn  
 Number of Key

### C. Merge Techniques

Merge use a full "GO" to insert and removal or forced strings are merged until a final string is formed.

#### (a) Two Way Merge

Method:

Keys are first compared in pairs placing each element in the pair in sequence. In the second pass two strings of two are merged to form a string of four. This is accomplished by comparing the first of each string and placing the lower in the larger string. Next the key last after the first compare is compared to the next key in the other string. Again the low is placed first. This is continued until the two strings from one string equal to the sum of the length of the original strings. The first pass of the 2-way merge is a simple exchange three pass.

Note:

The standard 2-way merge is probably the most used internal sort in commercial users. Merging is not affected by adverse sequences within the "GO". The programming is straight forward and number of keys are minimized. For a sort that is designed to handle a wide variety of situations the 2-way merge is a good technique.

#### Example 47:

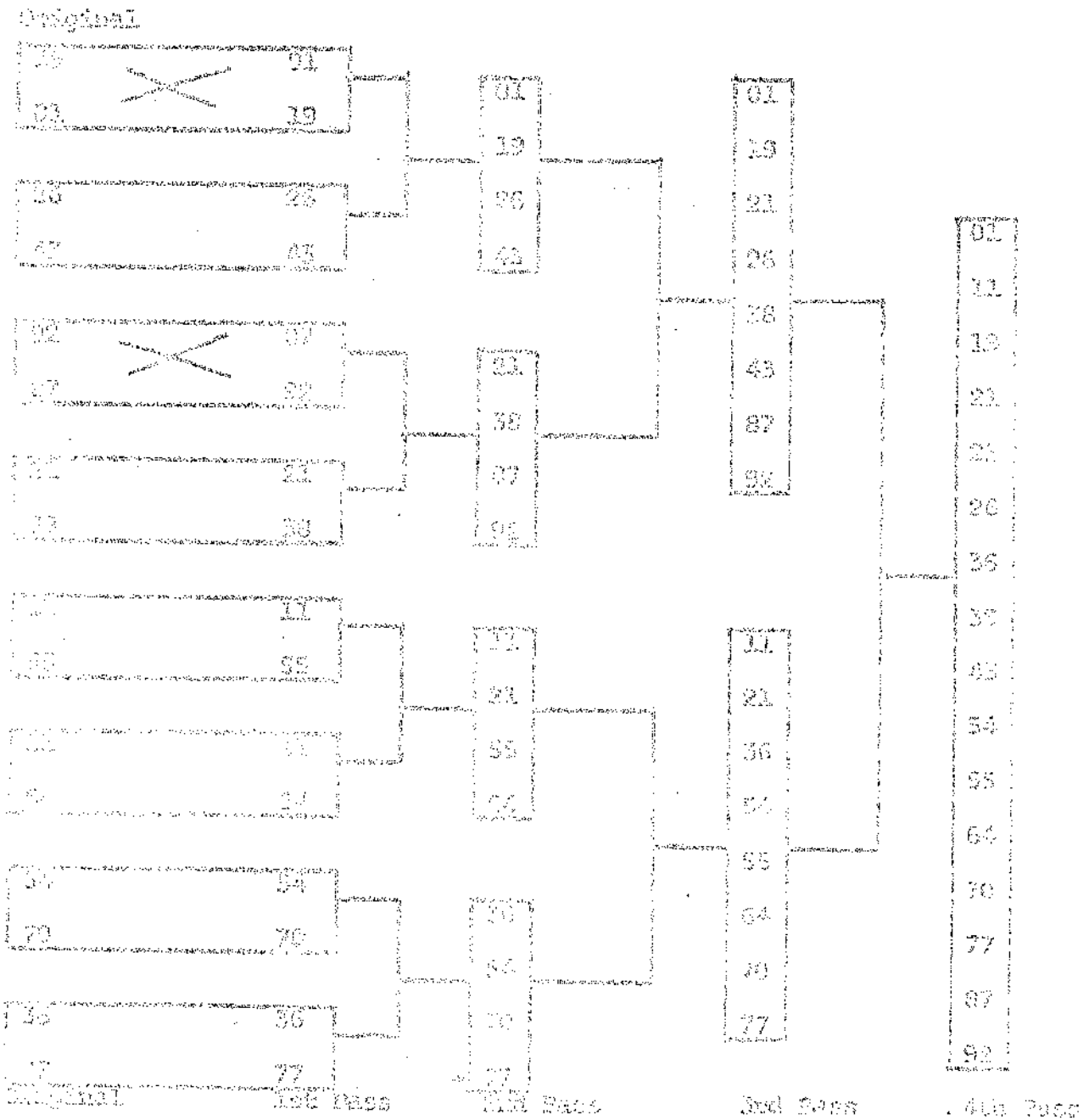
2-way merge builds strings of 2 using a single exchange then these strings are merged. The third set of strings of 2 is used.

Comparisons	Result String
1. 11 (1st) to 22 (2nd)	11
2. 33 (1st) to 44 (2nd)	33
3. 55 (1st) to 66 (2nd)	55
4. - Copy 66 (2nd)	66

112 Comparing proceeds in a similar manner.

Example 77

HEAVY MERGE



## 4) Variable String Merge

Method:

Instead of the exchange used in the first pass of the 2-way merge, the variable string merge sorts the natural sequences in the ascending or descending sequence. This ability is somewhat dependent upon the natural abilities of the computer used. When the strings are first merged, all remaining merge passes operate in identical manner except the strings to be merged are not necessarily equal in length.

Use:

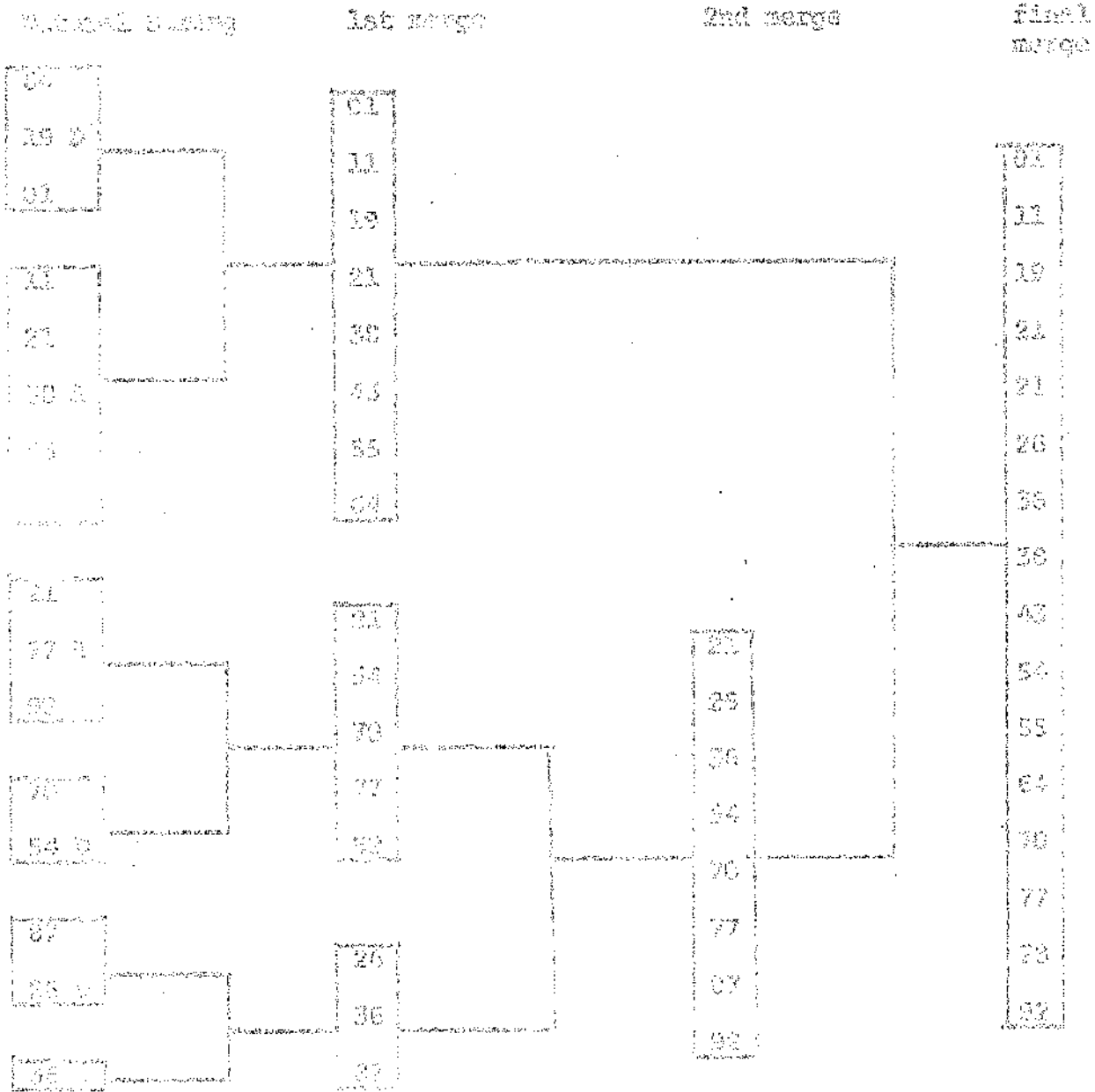
The use of this technique will take advantage of any natural sequence within a 70% group of records from input. If the 70% of records is in either ascending or descending sequence, it will be written immediately. Taking advantage of any natural string can give a significant time improvement. In the worst situation it is the same as the 2-way merge.

Example #7:

The string used in the other examples has been changed to better show the variable string merge. The first sequence (54, 19, 01) is placed in descending. The next (21, 21, 38, 43, 43) ascending. The 19 is placed in that it is a string of 1. The sequence involved to merge the above two strings are:

Sequence	Result String
01 (1901) to 11 (0101)	01
19 (1901) to 11 (0101)	11
19 (1901) to 21 (0101)	19
38 (1901) to 38 (0101)	38
43 (1901) to 43 (0101)	43
43 (1901) to 55 (0101)	55
54 (1901) Copy	54

Exercise 10  
Merge Sort



## 11. Insertion Technique

These methods start with a sorted array and fill it by bring in records one at a time.

### (a) Insertion

Method:

The entire array is initialized with a high pad (99) to save the operation of keeping a count in the number of records in array at any point time. A count must be used to eliminate an overflow of array.

Each record is compared to the records in the array area in sequence. When a low compare is found the remaining array is shifted one position to the right and the new record inserted at this point.

Code:

This version is a single register forward approach to a bubble sort. Probably the easiest of all techniques to program. The array can be entered in almost all other terminals. If dealing with a reverse sequence the number of compares is equal to  $n^2$ .

Example #0:

Row (10) compares low to (99) shift right and insert at first position.  
Row (01) compares low to (1) shift right and insert at first position.  
Row (25) compares high to (01) and (10) and low to (99) etc.



Input	Output
19 19	19 19 (99)
19 19	19 19 (99)
26 26	26 26 (99)
43 43	43 43 (99)
58 58	58 58 (99)
73 73	73 73 (99)
88 88	88 88 (99)
103 103	103 103 (99)
118 118	118 118 (99)
133 133	133 133 (99)
148 148	148 148 (99)
163 163	163 163 (99)
178 178	178 178 (99)
193 193	193 193 (99)
208 208	208 208 (99)
223 223	223 223 (99)
238 238	238 238 (99)
253 253	253 253 (99)
268 268	268 268 (99)
283 283	283 283 (99)
298 298	298 298 (99)
313 313	313 313 (99)
328 328	328 328 (99)
343 343	343 343 (99)
358 358	358 358 (99)
373 373	373 373 (99)
388 388	388 388 (99)
403 403	403 403 (99)
418 418	418 418 (99)
433 433	433 433 (99)
448 448	448 448 (99)
463 463	463 463 (99)
478 478	478 478 (99)
493 493	493 493 (99)
508 508	508 508 (99)
523 523	523 523 (99)
538 538	538 538 (99)
553 553	553 553 (99)
568 568	568 568 (99)
583 583	583 583 (99)
598 598	598 598 (99)
613 613	613 613 (99)
628 628	628 628 (99)
643 643	643 643 (99)
658 658	658 658 (99)
673 673	673 673 (99)
688 688	688 688 (99)
703 703	703 703 (99)
718 718	718 718 (99)
733 733	733 733 (99)
748 748	748 748 (99)
763 763	763 763 (99)
778 778	778 778 (99)
793 793	793 793 (99)
808 808	808 808 (99)
823 823	823 823 (99)
838 838	838 838 (99)
853 853	853 853 (99)
868 868	868 868 (99)
883 883	883 883 (99)
898 898	898 898 (99)
913 913	913 913 (99)
928 928	928 928 (99)
943 943	943 943 (99)
958 958	958 958 (99)
973 973	973 973 (99)
988 988	988 988 (99)
1003 1003	1003 1003 (99)

Example #9  
 Iteration

## (2) Binary Search

### Method:

The binary search technique is used as the search method in the framework of the Insertion Method (II, D, 1). The Regenerative technique (II, F, 1) is also involved in the example.

To explain this technique, a full [G] in sequence is assumed and we are adding records to G and taking the low records from "G".

In contrast to the compare method shown in insertion, the binary search with each compare eliminates half of the remaining records in "G". The  $\frac{G}{2}$  record is compared first. The  $\frac{G}{4}$  or  $\frac{3G}{4}$  records next depending on the result of the first compare. This process is continued until the proper location "G" is found. The low record is then placed in output and the "G" shifted to the left. The new record is inserted into its proper location.

If the new record is found to be less than the first record in G but higher than the previous output record, it can be placed in output immediately, leaving G intact. The effect of a sequence break record will be explained in the Regenerative technique (II, F, 1).

### Use:

The binary search is a compare technique to be used when finding the sequential location of a record in a group of records. As a sort technique, it uses less compares to find a location but will usually involve more program steps to write. If it is used to initially fill "G", the difficulty of a varying "G" size is encountered.

### Example #10:

"G" contains 9 records in sequence. 55 is next record. 55 is compared to 25 ( $\frac{G}{2}$ ) high. 55 is compared to 45 ( $\frac{3G}{4}$ ) high. 55 compared to 87 ( $\frac{7G}{8}$ ) low. 01 is placed in output and records in posi-

2, 3, 4, 5, 6, and 7 are shifted left one position. The 55 is inserted in position 7. This process is continued until a sequence break forces either a decrease of G size as a Regenerative or the records remaining in G written out and new string started with the sequence break record.

Example #10  
Binary Search

C = Compare    L = Low  
H = High       E = Equal

IP	"G" Positions									OP
	1	2	3	4	5	6	7	8	9	
	01	11	19	21	26	38	43	87	92	Full "G"
55					(1) C/H		(2) C/H	(3) C/L		Shift left from 71
	11	19	21	26	38	43	55	87	92	
21			(2) C/E		(1) C/L					Shift left from 71
	19	21	21	26	38	43	55	87	92	
60					(1) C/H		(2) C/H	(3) C/L		Shift left from 71
	21	21	26	38	43	55	64	87	92	
54					(1) C/H	(3) C/L	(2) C/L			Shift left from 51
	21	26	38	43	54	55	64	87	92	
70					(1) C/H		(2) C/E	(3) C/L		Shift left from 71
	26	38	43	54	55	64	70	87	92	
28	(4) C/E	(3) C/L	(2) C/L		(1) C/L					Replace 1
	38	38	43	54	55	64	70	87	92	
27					(1) C/E		(2) C/H	(3) C/L		Shift left from 71
	38	43	54	55	64	70	77	87	92	

### (3) Address Calculation

#### Method:

In the assignment phase a calculation is made to develop a factor to be used in the address calculation. This factor (F) is developed by dividing the range of keys by "G".

$$\frac{\text{Range}}{G} = F$$

In saving a "G", each key is divided by "F" to develop an effective address. This address is a relative position in "G". (This step need not be executed for all records before the next starts, but can be executed as records are received from input). Starting at first of "G" keys are placed in their address position until the situation arises that a key has an address that had previously been filled. The new record is then compared to the resident record and the lower placed in the address position. Sequentially, all adjacent filled addressed positions are compared to the high of the tree. Comparing continues until a higher is found. The remaining adjacent are shifted down one and the key is then inserted.

This process continues until the last key is placed in the "G" area.

#### Limit:

The address calculation method's value is to a large degree dependant on the distribution of the file to be sorted. The element that in most cases is considered good distribution (close to ascending or descending) can be very undesirable. Take, for example, a large file major and intermediate keys in sequence with many duplicate records only the minor field contains branches. In this case every record in a particular "G" group could develop the same address, completely negating the effect of the calculation.

A good file for this method is one that is randomly distributed. i.e.: for any one "G" most keys develop unique addresses.

A major part of this approach is the extended use of the divide command. There is a great difference in the speed of divides between techniques. Also, do the keys in the file, contain alpha? If they do, division is difficult if not impossible.

This method, for random records uses very few compares as compared with other techniques.

(3) Address Calculation (continued)

Example #14:

The range (00 to 99) is divided by G size (16) giving the resultant factor of (6).

The key is divided by 6 to obtain an address. In division, no remainder is used. The 19, 01, 26, 43, 92 are placed in address without conflict. The 21 with a repeat of the address 3 causes the first compare. The 19 already in address 3 is low and remains the 26 (higher than 21) in address 4 is shifted to address 5 and the 21 placed in address 4. The stops indicate positions where shifts must occur. All keys in "G" and the shifts are shown at this point.

Example #11

<u>Original</u>	<u>Address</u>		<u>1st</u>	<u>2nd</u>	<u>3rd</u>	<u>4th</u>	<u>5th</u>	<u>6th</u>
19	3	(1)	01	01	01	01	01	01
01	1	(2)		11	11	11	11	11
26	4	(3)	19	19	19	19	19	19
43	7	(4)	26	21	21	21	21	21
92	15	(5)		26	21	21	21	21
87	14	(6)		38	26	26	26	26
21	<u>1st Stop</u> 3	(7)	43	43	38	38	36	36
38	6	(8)			43	43	38	38
11	2	(9)		55	55	54	43	43
55	9	(10)				55	54	54
21	<u>2nd Stop</u> 3	(11)			64	64	55	55
64	11	(12)				70	64	64
54	<u>3rd Stop</u> 9	(13)					70	70
70	12	(14)	87	87	87	87	87	77
36	<u>4th Stop</u> 6	(15)	92	92	92	92	92	87
77	<u>5th Stop</u> 13							92
	<u>6th Stop</u> Final							

## E. Selection Technique

Selections start with a full G and pick one record at a time in desired sequence.

### (1) Quadruple Selection

Method:

"G" is first divided into sections of 4 keys. G sizes of 4, 8, 16, 32, 64 etc. work best. The lowest of each group of 4 is picked. The lowest of 4 groups is then picked etc. until the lowest in the "G" is found. Picking the lowest is called initializing "G" i.e.: that is to pick the low of all groups of 4 through all levels. After a low is found the program goes back to group from which the low was chosen and picks the new low of the group. Only the compares affected by the new pick are executed to find the next low.

Uses:

The major advantage of this method is that once "G" is initialized (the first low found) all following selections are made with a minimum of compares. Bad sequence has no effect upon the time or number of steps.

Example #12:

The lows selected in initialization are 01, 21, 11 and 36, from these the 01 is selected as the low. The 01 is replaced by the next low in the first section (19) and the second selection is repeated (19, 21, 11, 36), from which the (11) is selected. The (11) is then replaced by the 21 from the third set (19, 21, 21, 36). From this the 19 is picked as the third pick. This process is continued until all records from all quads have been picked. In example #12 the selections are arbitrarily divided into packs, 1-5 are shown in the first column, 6-10 in the second column and 11-15 in the third. The numbers in circles indicate the keys picked. Keys picked in the first column are eliminated in the second etc

Example #12  
Quadratic Selection

<u>Original 1 - 5</u>		<u>6 - 10</u>		<u>11 - 16</u>		<u>Output C</u>
(3) 19		--		--	--	01
(1) 01	01	--		--		11
26	19	(6) 26	26	--		19
43	26	(9) 43	43	--		21
						21
92		92		(16) 92	07	26
87	21	07	38	(15) 07	92	36
(4) 21	38	--	87	--		38
30		(8) 38		--		43
	01 11 19 21 21		26 36 38 43 54		55 64 70 77 87 92	54
(2) 11						55
55	11	55	55	(11) 55	55	64
(5) 21	21	--		--	54	70
64	55	64		(12) 54		77
						87
54		(10) 54		--		92
70	36	70	36	(13) 70	70	
36		(7) 36	54	--	71	
77		77	70	(14) 71		



## (2) Replacement Selection (Tournament Sort)

Method:

"G" is set to a number that is a power of 2 i.e. 2, 4, 8, 16, 32, 64 etc. G is filled with records. The first winner is found in the initialization of G. Pairs of records are compared and the address of the winner is placed in the first level table. Pairs of level 1 winners are compared and the winners addresses are placed in the level 2 winners table. This process is continued to standard tournament style until the winner of the final compare is placed in output.

The winner's address is used for the next record from input. The portion of the tournament affected by this change is played (one compare at each level) to find the second winner.

The input record is always checked with the last output record. If the input record would cause a sequence break an indicator is set causing a "bye" in this set. When the indicators are set on all records the string is completed and processing of the next output string starts.

After the first winner is found the number of compares to find the next is equal to the number of levels in the tournament.

$$n = \text{number of compares} \\ 2^n = G$$

Records are only moved from input to tournament and from tournament to output. While tournament is progressing they hold their same position in memory. The record address is used and compares made through indexing.

Notes:

This technique requires a small amount more memory for winners tables and program steps, but the low number of compares necessary to find each winner places it in strong contention as one of the fastest techniques. Sequence does not adversely affect the number of compares but it does affect the string length generated. When these advance techniques do occur other regenerative techniques are affected in the same manner.

Using random records string lengths averaging 2G are formed. When a complete reverse sequence occurs string length is equal to G or G-1, depending on whether the new record is brought in before the first winner is placed in output or not. If all records are within G records of their sequence in output, a single string will result.

Moore, Martin A. "Internal and Tape Sorting Using the Replacement-Selection Technique." ACM Sort Symposium November 1952

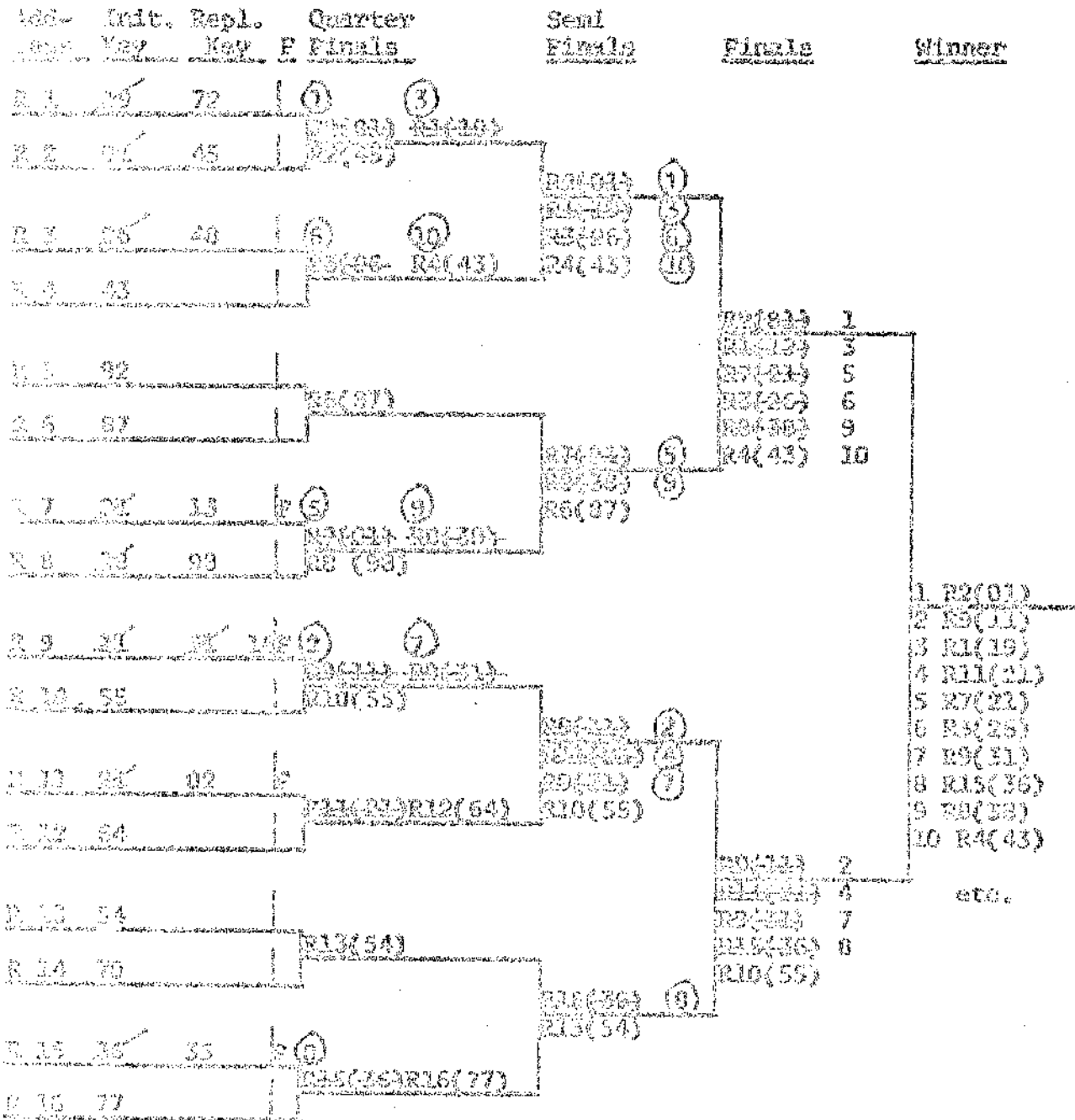
(3) Replacement Selection (Tournament Sort) continued

Examples 13 A, 13 B, 13 C, 13D:

Due to the difficulty of picturing the dynamic aspects of this method several examples are used. Example #13A shows the tournament through the outputting of 10 records. No new input record has been entered to replace R4. The tournament (string) ends when all record positions are forfeited (F). To show the steps in the development of example #13A, examples #13B, example #13C, example #13D are used. Example #13B shows the status of the tournament on initialization. Quarter final, semi-final, and finals tables are filled. R(01) is selected as the winner (first output record). Example #13C shows the number of compares to determine the second winner R(11). The key (45) has replaced (01) in R2. Example #13D shows the second forfeit (F) record C + 5 (the fifth record in after C was initially filled) a (13). The last output record at this point was R(21) or higher than the input (13), therefore a forfeiture (F) is caused and the tournament continues.

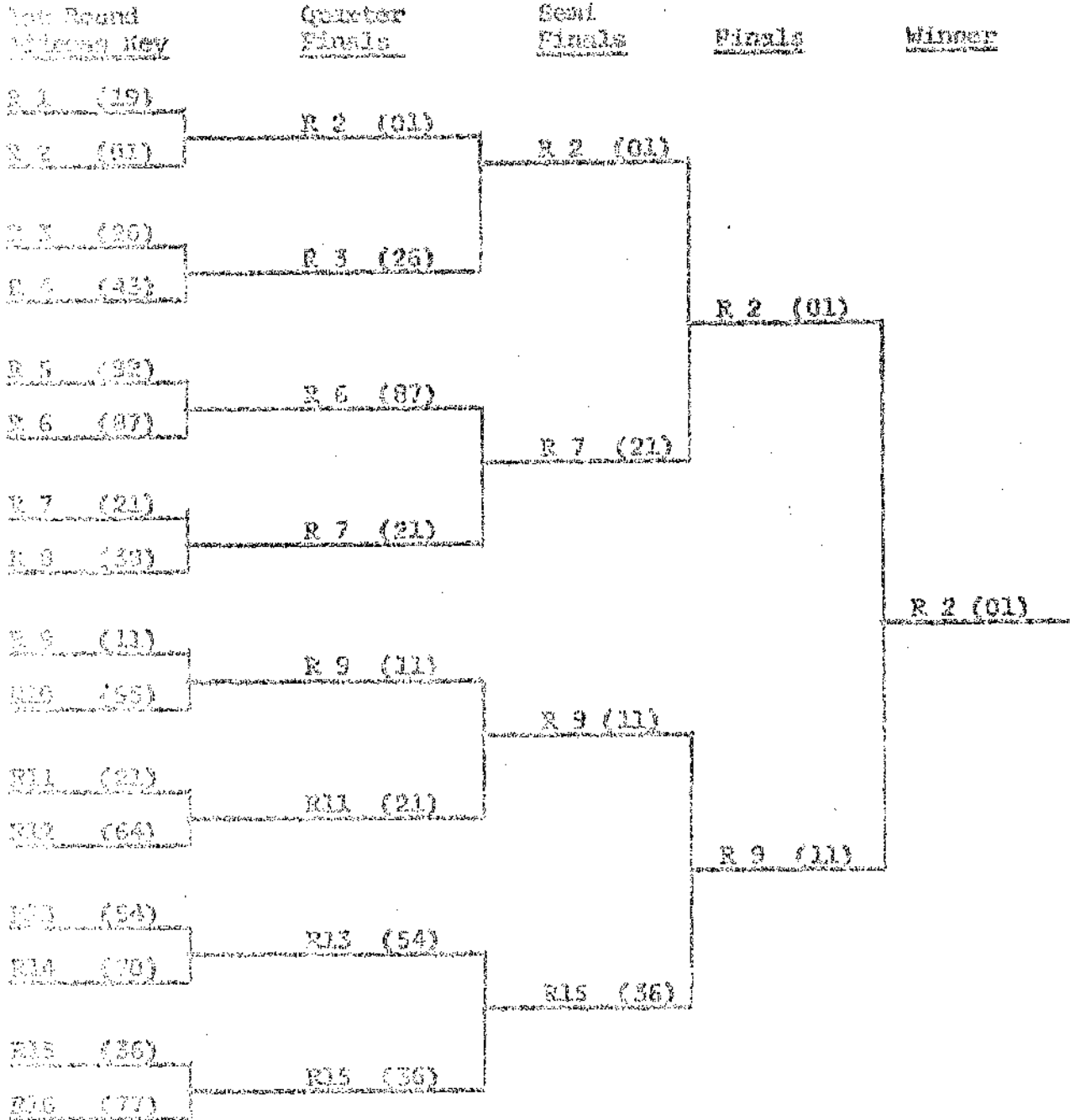
The end of this string is not shown, but would occur when the last of the record addresses becomes a forfeiture. Tournament is again initialized to start the next string.

Example #13A  
REPLACEMENT - SELECTION, Tournament Sort

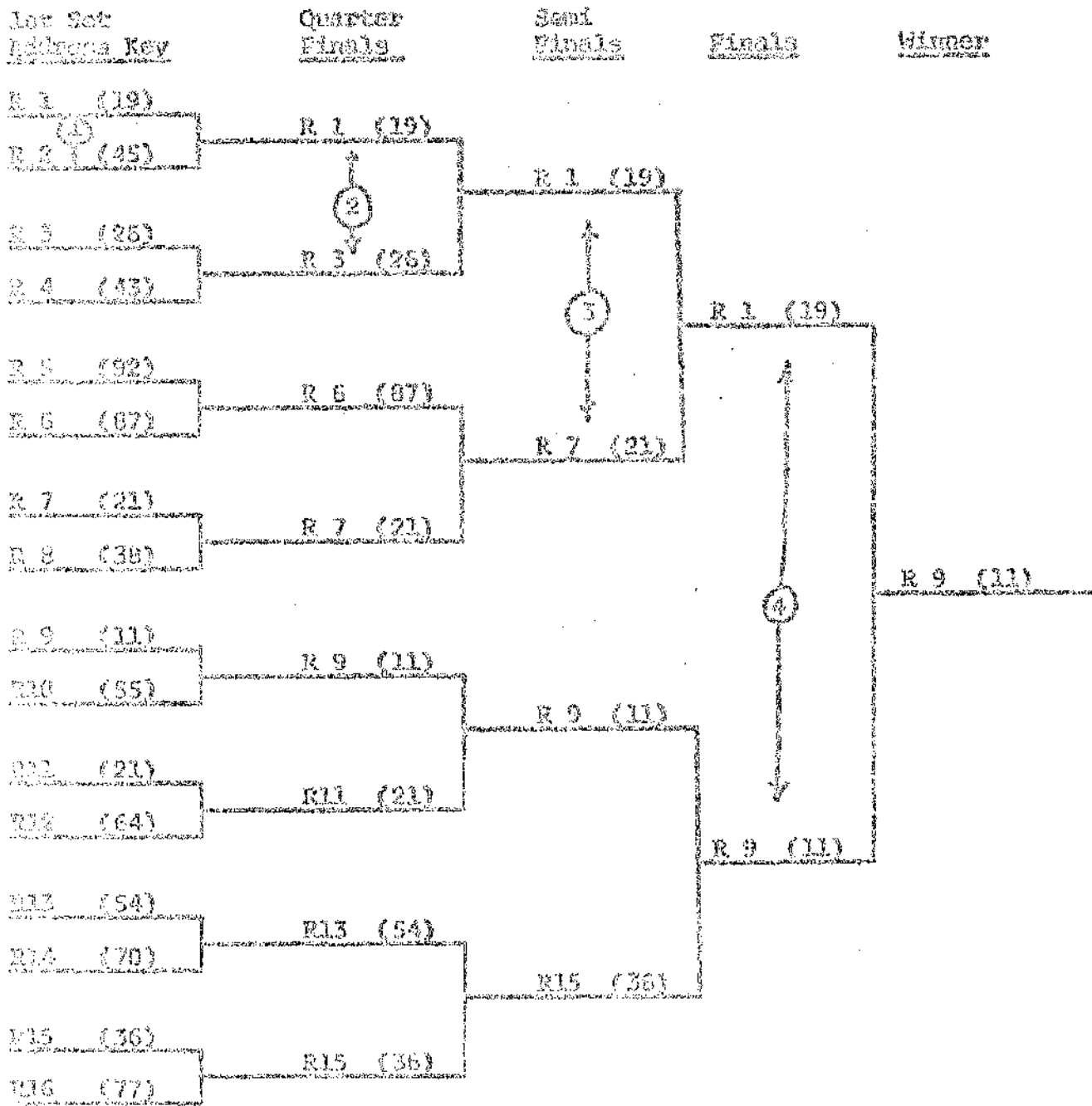


Input after initialization 45, 31, 72, 02, 13, 48, 16, 35, 98 etc.

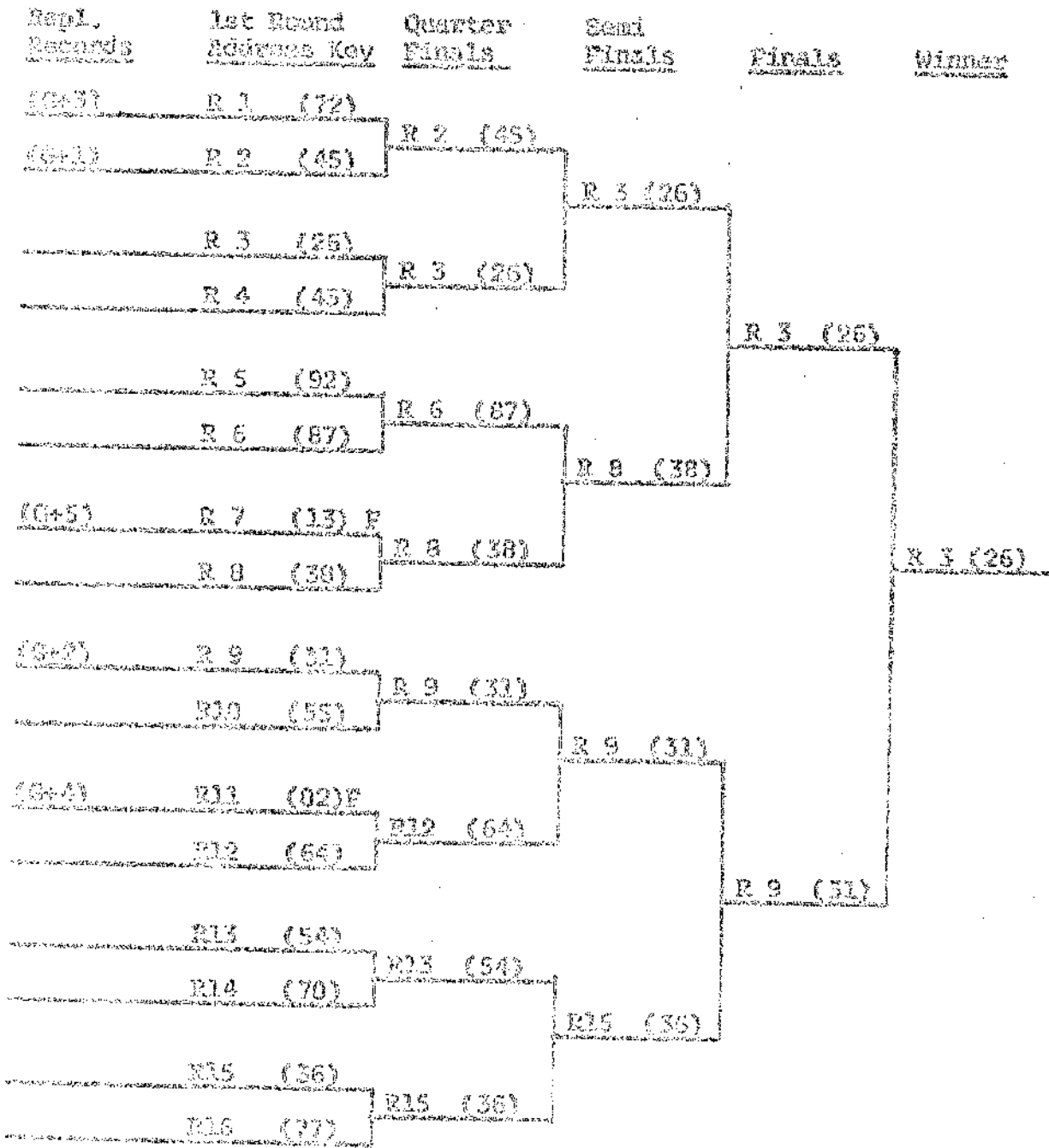
Example #133  
REPLACEMENT - SELECTION, Tournament Sort  
Initialization of Tournament



Example #130  
 REPLACEMENT - SELECTION, Tournament Sort  
 Play to Determine 2nd winner - New record is 45 in Record 2



Example #130  
 REPLACEMENT - SELECTION, Tournament Sort  
 Status of Tournament at Input of (6+5) Record



## F. Regenerative Technique

This is not a basic technique as the others but can be used in conjunction with the insertion and selection techniques to develop strings longer than  $G$ . It does not apply to the exchange, Radix or merge techniques.

### (1) Regenerative

The regenerative technique is not a sorting method in itself but a method of expanding several other methods to develop a string of records longer than the " $G$ " size.

#### Methods:

$G$  records are sorted into sequence. The next record is placed into the sequence making room for it by placing the low record in output. When an input record causes a sequence break it is placed in the first position of  $G$  and the size of  $G$  is decreased by 1. The next sequence break record is properly positioned and again  $G$  is decreased by 1. The string is terminated when  $G$  is equal to 0.

#### Use:

This technique is used to develop strings longer than  $G$ . The insertion, binary search, replacement selection or quadratic selection techniques can utilize this method. In a reverse sequence the length of the string is  $G + 1$  but dealing with random files the string length averages about  $2G$ .

#### Example #14:

The small  $G = 4$  was selected to show a complete cycle. The first four records were placed in  $G$  by the insertion method.  $G$  is 01, 19, 26, 43 when the regenerative procedure starts. The 92 is placed in  $G$  while the 01 is placed in output. When the first 21 is the input record it is found that it is less than the first record but higher than the last output so it is placed in output immediately with  $G$  unchanged. The 11 key causes the first sequence break.  $G$  is now equal to 3. At the second 21 key the  $G$  size becomes 2. The addition of the 70 key causes  $G$  to be depleted and the string ends.  $G$  reverts back to 4 and the process continues.

Example #14  
Repetitive

<u>Input</u>	<u>C = 4</u>	<u>Output String</u>
19	19	
01	01 19	
26	01 19 26	
43	01 19 26 43	
92	19 26 43 92	01
87	26 43 87 92	19
21	26 43 87 92	21
38	38 43 87 92	26
11	11 43 87 92	(2.5 C) 38
55	11 55 87 92	43
31	11 21 87 92	55
64	11 21 87 92	64
87	11 21 56 92	87
70	11 31 56 70	<u>92</u>
36	21 36 56 70	11
77	36 56 70 77	21
	(At least 4 more to fill this sequence)	36
		56
		70
		77



## Section III

### MERGING

Merging is used in the third phase of a generalized sort. Strings developed in the preceding phase are brought together forming longer strings with each pass. Input/output devices are implied as opposed to section II where all manipulations were within memory. In all examples numbers refer to strings and are not keys.

#### A. Balanced Merge

As the name implies, balanced merge uses an equal number of input/output devices on either side of the merge network. A two channel computer, exemplifies this. On a pass, reading is performed on one channel, writing on the other. The desire on this arrangement is to obtain complete overlap of reading and writing. Balanced merging can be accomplished in any order (n) depending upon the number of available I/O devices.

To determine the proper order of merge, several considerations are important:

(1) block size, (2) memory size, (3) relative input/output speed to calculation speed, (4) size of control field, (5) size of record.

The combination of these factors will determine the most efficient order of merge.

#### (a) Balanced Binary Merge

Only one example of balanced merging is shown due to complete similarity between merges of different orders. It is reasonably safe to assume that the higher the order of merge the fewer passes it will require to complete the merge phase. This statement can be misleading when dealing with specific generalized sorts. Using a 3-way merge as opposed to a 2-way merge in the preceding phase will have two output file requirements added. This could make a significant difference in the string lengths developed. In many situations it is possible for a 3-way merge to beat a 2-way merge. Over the entire spectrum of sorting volumes and record lengths, the 3-way is superior, but in certain cases the 2-way is significantly faster.

(1) Balanced Merge Merge (continued)

Method:

The preceding phase places a group of strings on each of the devices. During a merge pass one string each from all input devices comes together to form one string on the output device. (See example 14A). The next set of strings (second string on each device) is merged into a single string on the next output device. After the output string is placed on the last output device, the next is placed on the first. A pass is completed when all input strings have been placed in output. The number of strings placed on the output is calculated:

$$\frac{\text{Input strings}}{\text{order of merge } (n)} = \text{output strings}$$

The second pass starts reading the strings prepared by the preceding pass. Passes continue until  $n = 1$ .

The final merge pass is often designated as a separate phase to facilitate late sort modifications. The last pass occurs when there is one string or less strings on each of the input devices (Phase 3 of example 15B).

Example 15 A:

In example 15 A the transition is made from keys to strings. An unrealistic string length of 4 is used to have strings small enough to show the effect on a single page.

Strings 1, 2, and 3 are brought together comparing the 01, 21 and 11. The 01 is low and placed in output, the 01 is replaced by the 19 and the low of 11 outputed next. Example 15 A shows computer forming string A on output from S-1, S-2, and S-3 on input.

Example 15 B:

Example 15 B shows how strings 1, 2, and 3 on input form string A on output. Strings 4, 5, and 6 form string B, etc. When the final output (page six) is used, the next string "B" is placed on the first output (page four).

(1) Unbalanced 3-way Merge (continued)

Example 15 C:

Example 15 C shows the merging of 30 strings from the preceding phase down to one string in four passes. Notice that at the end of the first pass there are  $\frac{3(30)}{2} = 45$  strings left.

$$\frac{3(30)}{2}$$

Pass two shows how the number of remaining strings does not remain equal when "n" does not divide evenly. After the merge of the ten strings, four remain. The 3 string was in effect copied. The inefficiency shows more drastically in the final pass where the strings I, II, and III are merged with string IV. Strings I, II, and III represent 27 original input strings where the string I or IV represents only 3. The last three strings in effect cause two extra passes.

If the order had been 4-way, the result would be

Pass 1	-	30 to 8
Pass 2	-	8 to 3
Final	-	3 to 1

If a 5-way merge were used

Pass 1	-	30 to 6
Pass 2	-	6 to 2
Final	-	2 to 1

If a 6-way merge were used

Pass 1	-	30 to 5
Final	-	5 to 1

Probably the most effective, assuming the same string lengths, would be the 6-way in two passes. This discussion will be continued in the variable order of merge, section III C (2).

Example 15 A

COMPARISON OF PALAPLAN MEMO

Input		Compares			Output
key from string		S1	S2	S3	key from string
01	1	01	--	--	
21	S2	01	21	--	
31	S3	01	21	11	
Computing Starts:		-01-	21	11	01 - S1
19	- S1	19	21	-11-	11 - S3
21	- S3	-19-	21	21	19 - S1
26	- S1	26	-21-	21	21 - S2
33	- S2	26	33	-21-	21 - S3
45	- S3	-26-	33	33	26 - S1
43	- S1	43	-33-	33	33 - S2
43	- S2	-43-	33	33	43 - S1
55D	- S1	--	33	-33-	33 - S3
64	- S3	--	33	-33-	64 - S3
END	- S3	--	-33-	--	33 - S2
32	- S2	--	-33-	--	33 - S1
END	- S2				END OF STRING

Example 15 D

BALANCED MIXER (Keys to Strings)

Input			Output		
Tape 1 St. key	Tape 2 St. key	Tape 3 St. key	Tape 4 St. key	Tape 5 St. key	Tape 6 St. key
1 — 01 19 26 33	2 — 21 38 57 92	3 — 11 21 55 64	01 11 19 21 21 A — 26 38 43 55 64 87 92	02 13 16 31 36 B — 43 48 54 70 72 77 98	23 29 35 65 51 C — 58 64 69 71 79 83 91
4 — 36 54 70 77	5 — 13 45 48 72	6 — 02 16 31 80	14 15 21 21 31 33 D — 51 66 70 80 84 96	etc.	
7 — 38 45 71 83	8 — 25 29 54 69	9 — 51 58 79 91			
10 — 15 32 46 80	11 — 31 33 70 86	12 — 14 21 21 84			

etc.

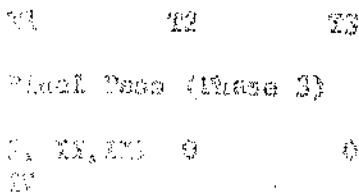
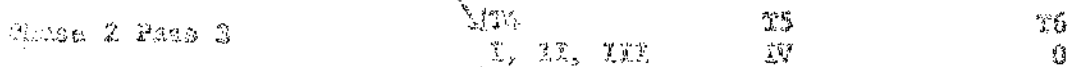
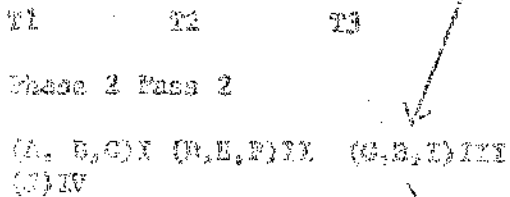
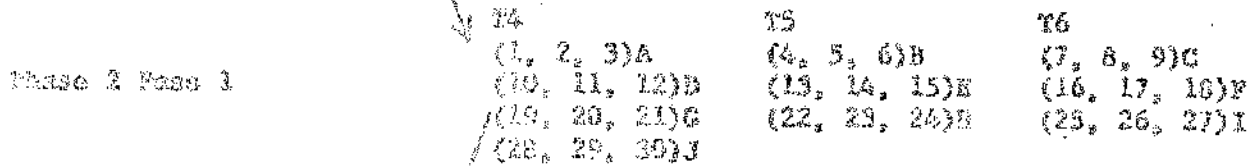
\* St. = String

Example 15 6

DAMAGED 3-WAY MERGE (Merino)

Output Phase 1 (arrives)

T1	T2	T3	T4	T5	T6
1	2	3	0	0	0
4	5	6			
7	8	9			
10	11	12			
13	14	15			
16	17	18			
19	20	21			
22	23	24			
25	26	27			
28	29	30			



## ii. Polyphase Merge (DeBelenand)

The polyphase merge is the name given to any unbalanced merge technique. The polyphase methods follow the Fibonacci series. This series can be represented as:

$$F_n = (F_{n-1}) + (F_{n-2})$$

or in simple numbers any term is the sum of the preceding two terms, i.e., 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610 etc.

Fibonacci developed this series in the early twelve hundreds, experimenting with the breeding of mice. Many have expressed doubt as to what the relationship between mice and nest is, but the series seems to work. The only place where the series is used unmodified is a three tape sort. Polyphase sorts use a modification of this series on higher order merges to determine the distribution of strings during the execution of the internal sort phase.

### (3) Cascade Merge

The cascade merge was probably the first of the polyphase techniques to be implemented. The outstanding feature is the utilization of the read backwards feature to completely eliminate rewind time. The other effect is a higher merge order.

A 3-way balanced merge requires six tapes, a 3-way cascade requires four.

#### Method:

Strings are distributed in a prearranged manner to approximate the series. Reading backwards, a 3-way merge is executed for the number of strings on the smallest tape. When this tape is exhausted, a 2-way merge is executed with the output on the tape just completed. At the next read completion, a copy is executed to complete a pass.

#### Example 16:

The strings are distributed TA-14, TB-11, TC-6, TD-3.

5 strings are merged to TD (3-way)

5 strings are merged to TC (2-way)

3 strings are merged to TB (1-way - copy)

(1) Greedy Merge (Continued)

At the start of the second pass TA-0, TB-3, TC-5, TD-6.

3 strings are merged to TA (3-way)

2 strings are merged to TB (2-way)

1 string is merged to TC (1-way - copy)

At the start of the third pass TA-1, TB-2, TC-1, TD-0.

1 string is merged to TB (3-way)

1 string is merged to TC (2-way)

1 string is merged to TD (1-way)

At the start of the fourth pass TA-0, TB-1, TC-1, TD-1.

The rule here is the same as with balanced merges.

The final pass starts when not more than one sequence remains on any of the input tapes. This final pass merges one string from each tape to the final string on TA.



Example 16

CASCADE

3-Way Merge on 4 Tapes

	TA 16 Rd	TB 11 Rd	TC 6 Rd	TD 0 Wr	
1st pass	8 Rd 3 Rd 0	5 Rd 0 Wr 3	0 Wr 5 5	6 6 6	3 way 2 way copy
2nd pass	Wr 3 3 3	Rd 0 Wr 2 2	Rd 2 Rd 0 Wr 1	Rd 3 Rd 1 Rd 0	3 way 2 way copy
3rd pass	Rd 2 Rd 1 Rd 0	Rd 1 Rd 0 Wr 1	Rd 0 Wr 1 1	Wr 1 1 1	3 way 2 way copy
4th pass	Wr 1	Rd 0	Rd 0	Rd 0	

(3) ~~Exhaustive Merge~~ (used in Sort 2, 1401) 3-way on four tapes

Method:

Internal sort (phase 1) is executed in the normal manner putting strings out on two output tapes.

An adjustment phase is executed next distributing the strings so that they can be merged down to the next lower number of strings entry in the polyphase table. At this point the polyphase sort starts.

In this method read backwards is not used and tapes are rewound. Three tapes are merged to one until the smallest of the input tapes is depleted. The output tape and the input tape are both rewound and 3-way merging continues. The table is constructed so that there will be one string on each of three tapes for the final pass.

Example 17:

All numbers in example how remaining strings:

- 1 Shows the output of internal sort. 54 strings T3-28, T4-27.
- 2-4 Copy 11 strings to T1; 6 from T4 and 5 from T3. Copy 5 strings from T4 to T2. Tapes are now set to merge down to the next lower number of strings in the table.
- 5 Eight strings are merged (2-way) T3 and T4 to T1.
- 6 Eight strings are merged (3-way) T1 and T3 and T4 to T2.
- 7 The 31 strings remaining are distributed according to table T1-11, T2-13, T3-7. This merge pass leaves the tape with 17 strings (4,6,7), the next lower entry in the table.
- 8-11 The merging process is continued, each step merging the number of strings on the smallest tape.

At first glance the complication in the adjustment phase and the complexity of keeping track of the unbalanced merge would seem to negate the saving. In the running of many samples with identical input, Sort 2 (four tapes) polyphase compares favorably in running time with Sort 2, 3-way merge (six tapes). The results of high-way merges using like processes would be interesting to study. The indications are that the polyphase, even without the read backwards, is superior to the balanced

Example 17

POLYPHASE MESSAGES

START COORDINATION

ORIGINAL MESSAGE

1 Develop 93 strings

	T-1	T-2	T-3	T-4	
S	S OP RW	S OP RW	S OP RW	S OP RW	
OP	R X O		OP W X	OP W X	

Adjustment Tables

- 2 S (3) from T4 to T2
- 3 S (6) from T4 to T1
- 4 S (5) from T3 to T1
- 5 U (8) from T3, T4 to T1
- 6 H (8) from T1, T3, T4 to T2

OP	O		5 W	18		24 R	
5 S	6 W		5		18		16 R
5 S	11 W		5		23 R		16
4 U	19 W X	5			15 R		5 R
6 H	11 R		13 W X	7 R			0 R X

Copy Message

- 7 H (7) from T1, T2, T3 to T4
- 8 Y (4) from T1, T2, T3 to T3
- 9 L (2) from T2, T3, T4 to T1
- 10 M (1) from T1, T2, T3 to T2

7 H	4 R		6 R		0 R X		7 W X
8 Y	0 R X		2 R		4 W X		3 S
9 L	2 W X		0 R X		2 R		1 R
10 M	1 R		1 W X		1 R		0 R Y

Final Message Message

11 H (3) from T1, T2, T3 to T4

11 H	1 R X		1 R X		1 R X		1 W X
------	-------	--	-------	--	-------	--	-------

Mathematical table used in Polypphase

Total Strings	Distribution on Tapes		
	A	B	C
3	1	1	1
5	1	2	2
9	2	3	4
17	4	6	7
31	7	11	13
57	12	20	24
103	24	37	44
193	44	63	81
353	81	123	149
653	149	230	274
1201	274	423	504
2209	504	770	927

- S - Strings
- R - Read
- W - Write
- OP - Operation
- RW - Read (X)
- T - Tape
- (n) - Number of Strings
- C - Copy
- M - Message

## (4) Read Backward Polyhedra 2 or Unillating Polyhedra

### Method:

During the internal sort phase, records are written on output tape with strings in alternating sequence. The first string on one tape must be ascending (A), the next descending (B). Strings are then placed on tapes according to table or formula alternating between (A) and (B) sequence. The strings must still sort the table sequence. If strings fail between table entries, padding strings are used.

Tapes are read backwards and write for order (no rounds until after final merge). Strings are merged until the smallest tape is depleted. At this point this becomes the output and the just completed tape becomes part of the merge. The process is continued until the merge tapes have one string each.

It will be noted that with the read backward all (A) input to merge is written and in (B) sequence and all (B) input is merged and written in (A) sequence. This takes full advantage of the reverse read without having to copy to the reverse list.

A significant point about this method is that on each pass, except the first, only part of the file is passed. In fact, in the three passes shown in Sample 117, the large file is completely passed only once to the next stage reader. The sorting process is consistently good regardless of how it is done. The only difficulty is how to obtain a good internal balance as a multi-pass sort sequence. This problem seems minor. There is, however, a difficulty in modifications to phase 2. Phase 1 and 3 modifications would be no more difficult than in a balanced sort.

### Sample 117:

Strings 7-01, 6-12 and 5-02 enter the merge. Four strings are merged from 01, 02, 03 and 04, leaving 5-01, 2-02, 0-03, 1-04. The strings are merged from 01, 02 and 03 to 02, leaving 0-01, 0-02, 2-03, 1-04. The strings are merged from 01, 02, 03 to 02, leaving 0-01, 1-02, 1-03, 2-04 (the rearrangement and the final pass).

- D. L. Gilman, Minneapolis-Polytechnic, presented at the Association of Computing Machinery Symposium, November 29 and December 30, 1962.

Example 10

DEEP BACKSIDE POLYTRAP

45

CONTINUING RELEASE

Output of Phase 1

<u>T1</u>	<u>T2</u>	<u>T3</u>	<u>T4</u>
1-A	2-B	3-D	0
4-B	5-A	6-A	
7-A	8-B	9-B	
(7)10-B	(6)11-A	(6)12-A	
13-A	14-B		
15-B	16-A		
17-A			

Phase 2, pass 1

1-A	2-B	0	E(17,16,12) (D)
(2) 4-B	(2) 5-A		(4) F(15,16,9) (A)
7-A			G(13,11,6) (B)
			H(10,8,3) (C)

Phase 2, pass 2

(1) 1-A	0	I(7,5,11) (D)	J(17,16,12) (D)
		(2)1(6,7,6) (A)	(2) K(15,16,9) (A)

Phase 2, pass 3

0	(1) H(1,9,7) (C)	(1)1(7,5,11) (D)	(1) J(17,16,12) (D)
---	------------------	------------------	---------------------

Phase 2, pass 4

(0,1,8) (A)

### C. Random Access Techniques

With the advent of large high speed random access devices, some new aspects have been opened up in sorting. Some of these affect internal sorting almost as much as merging, but since the basic techniques used in internal sorting aren't affected, random access techniques are included under merging.

Random access devices eliminate the need for some sorting in cases where files are permanently stored in them. But in this discussion, it is assumed that the file is used as a scratch pad for files external to the disk that must be sequenced.

The seek remains an extremely important consideration. Seek time is in the same category as rewind time on tape as non-productive time unless overlap can be obtained. In one example of importance of seek time, the run time of a sort was decreased by 1/3 with the addition of an arm on the disk unit.

There remains a need in this area for more study to find better techniques to show the full potential of sorting on disk units.

#### (1) Tag Sorting

Tag sorting has been widely used in internal methods instead of moving the record in memory (this is strictly applicable to disk). The tag is formed by taking the control field and adding to it the disk address.

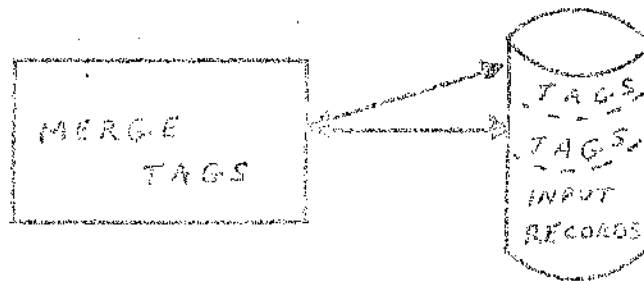
Phase 1:



1. Reads records from tape or file.
2. Places records on disk.
3. Takes control field from record, adds disk address and forms a small control record.
4. Builds a "G" from the tag records.
5. Writes "G" on disk file in maximum of five files.

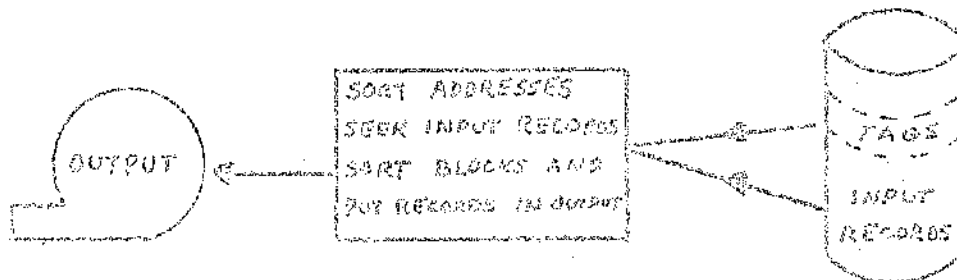
(1) Tag Sorting (continued)

Phase 2



1. Merges tag files from disk as if they were from tape.
2. Final phase 2 pass will strip control fields from tag records leaving the disk addresses only (or the user's option sort 3 can end at this point).

Phase 3 (optional)



1. Disk addresses are brought into memory in some multiple of the output blocking factor.
2. Addresses are then sorted into ascending sequences to minimize seek time.
3. Input records are then read from the disk and placed in the proper sequence, then written on tape or in disk memory.

The seek time in this method seems to almost negate the effectiveness of the technique. However, the implementation is on one of the slowest disk units. With the advancement of new equipment, this method may still be feasible.

## (2) Variable Range Order Disk Sort\*

### Example:

1410-1301 Sort is a partial example, however, order is determined by 1410 core size. The standard merge for this sort is a 12-way merge. This sort is as close to this technique as any implemented now. In many volume ranges, it compares favorably with the 7070 Sort 90.

### Considerations:

1. Core size (ability to handle input blocks).
2. Seek time and re-write time.
3. Convenient block size as specified by file considerations.

### Phase 1:

1. Would sort 8's determined by record length, file and memory consideration as large as convenient.
2. Strings would be written as if they were being placed on one tape.
3. All string locations placed in a table as to position in file and high and low values.

### Phase 2:

Using the information gained during phase 1, a formula, built around hardware considerations, could be solved for the most efficient order of merge. It is not necessarily a balanced merge; a polyphase could be very effective.

1. Table of sequences and address should be scanned and sorted to place all actual strings together.
2. When most efficient merge is determined, table should be broken to define merge files.
3. Formula should be evaluated prior to each pass to have proper order.

### Phase 3:

Could compare as a standard sort.

\* Toloney, J.R., Midwestern Region (publication unknown),  
August 1961, "A Sort Designed for the IBM 1410-1301 Disk Storage System (P140)"



## Section IV

### CONCLUSION

The sort of the future will probably look no more like our sorts today than a computer sort resembles a mechanical sort. The element that some future sorts will have is the use of multiple techniques in one sort package, i.e., instead of just picking the order of merge. The package will evaluate the specifications or parameters and decide what type of sort will produce the best results. Sort will be an operation (high level macro or token sort verb). The programmer will use the sort macro, and a specific tailored sort will be generated as a part of his processing job.

Sorts and sort techniques can be very difficult to evaluate. The random files that are used to test the effectiveness of sorts just don't exist in installations. It is recommended that Sort Sense be studied to show the pitfalls that arise when comparing sorts. Much of the analysis of sort techniques centers around computer or other criteria. I feel that the only true evaluation (including the computer and package) is the number of records sorted per computer dollar.

Sorting by itself is a non-productive though highly important aspect of data processing. The fastest sort in the world is no sort. This is the sort that the Systems Engineer figures out how to avoid. No reports to management are produced while sorting. Yet many reports are highly dependent upon sorts. With the acceptance of management by exception, the cause of sorting will receive another blow. Sorting will decrease in the amount of time spent in the data processing installation, but probably never be eliminated.

\* Part Seven, Information Regional Bulletin 57, May 18, 1961.

Section V

A GLOSSARY OF SORTING AND MERGING TERMS

ADDITIONAL PROGRAM - The set of instructions by which a generalized program modifies and completes the set of instructions which will be utilized in performance of one specific sort run.

ASCENDING SORT - A sort in which the final sequence of records is such that successive keys compare, "greater than or equal to".

1 - Internal sort blocking, unbuffered records between inter-record gaps on tape. 2 Input blocking, 3 output blocking.

BACKWARD READ - A feature available on some magnetic tape systems whereby the magnetic tape units can transfer data to computer storage while moving in a reverse direction. Normally used, if available, during the external sort phase to reduce rewind time.

BALANCED SORTING - See: (X/2)-way merging.

BIASED DATA - A distribution of records in a file which is non-random with respect to the sequencing criteria. Biased data may affect sorting time, depending on the technique used during the first pass of the data. Normally, it is desirable to take advantage of existing sequence bias (whether ascending or descending) during the first pass.

BRIDGE MERGING - A technique used in a sort program to merge strings of sequenced data. Given  $T$  work tapes, merging is performed at  $T-1$  on part of the data,  $T-2$  on parts of the data, and so on. Strings of sequenced data are distributed in a Fibonacci Series on the work tapes preceding each merge. The effective power of the merge varies between  $T-1$  and  $T/2$ , but in all cases is less than the power of the polyphase merge. CS: effective power of the merge.

CHECKPOINT - Also: restart point. The point at which a restart (or rerun) can be initiated. Memory, registers, and the position of tapes are recorded at this point. CS: restart point.

CANCEL TAPE, "CCTP" - An optional word in COBOL 61.

COALESCING - Sequencing a group of records by comparing the key of one record with another record until equality, greater than, or less than is determined.

GLOSSARY (continued)

COLLATING SEQUENCE - The sorting sequence; a description of the sort key for a file of records.

COLLATING SWITCH - A sort which uses a technique of continuous merging of data until one sequence is developed.

COMPARA LIMITED - See: program limited.

CONTROL CARD - A card which is used to specify the parameters for a sort.

CONTROL FIELD, CONTROL AREA FIELD - A continuous group of characters within a record which form all or part of the control word  $F_1, F_2, F_n$ .

CONTROL WORD - A word in the key.

CUBERIA - See: Sequencing cuberia. See: key.

CUMULATIVE SORT - A sort in which the final sequence of records is such that the successive keys compare "less than" or "equal to".

DESERIAL SORTING - A sort which uses a technique similar to working on substitution machines (e.g. IBM Sorter). The elapsed time is directly proportional to the number of characters in the sequencing key and the volume of data. Also "radix sort".

DISK SORTING - A sort program that utilizes disc memory for auxiliary storage during sorting.

DRUM SORTING - A sort program that utilizes magnetic drums for auxiliary storage during sorting.

EFFICIENCY INDEX OF THE INDEX - Equal to  $\frac{1}{n}$ , where  $n$  is the number of input elements and  $n$  is the average number of times each element of data is read.

FIBONACCI SERIES - a series where the current number is equal to the sum of the two preceding numbers: i.e., 1, 2, 3, 5, 8 and so on. Some sort programs distribute strings of data onto work tapes so that the number of strings on successive tapes form a Fibonacci series.

GLOSSARY (continued)

FIRST PASS CPU CODE - Computer instructions created by the programmer, in assembly or absolute form, which are executed by a sort during the first pass of the file after input program has been loaded but prior to execution of first pass sequencing instructions.

FIXED SIZE RECORDS - Delineatable file elements each of which has the same number of words, characters, bits, fields, etc. as variable-length records.

G - Area in memory used to store records into sequenced strings. In some sorts 2 "G" words are used. It is desirable to increase the string length to decrease the number of phase 2 passes (sometimes called RIB).

GENERATION SORT - A test program which will accept the introduction of programs at run time and which does not generate a program.

GENERATED SORT - A production program which was produced by a sort generator.

HASH TOTAL - A total taken on an arbitrary field of the record for control purposes only. The hash total taken during one pass may be compared to one taken during a previous pass to aid in detecting any mis-handling of records.

IBM LIMITED - See: Copy Limited.

INPUT PASS(n) - Topical processing a file in arbitrary sequence to be introduced into a sort/merge program.

INTERVIEW METHOD - See: Coding.

INTERMEDIATE PASS - Any phase of a merging operation which, because of the number of strings, or otherwise, does not reduce the file to a single sequenced string.

INTERMEDIATE PASS CPU CODE - Computer instructions created by the programmer, in assembly or absolute form, which are executed by a sort during the intermediate passes of the file after the execution of instructions has terminated in order to be executed for output of the selected records. They also be executed during the interval pass, but after the selection of records.

IRI - See: record.

INDEX/ANN (Continued)

KEY - Also, sequencing key; ordering, sequencing criteria. The fields in a record which determine, or are used as a basis for determining, the sequence of records in a file.

HEADER - A record, usually located at the beginning of a file, which contains information related to identification, operation, and control of the file or refers to a record which is part of the file itself.

LOAD DATA INTO PROGRAM - Computer instructions created by the programmer, in assembly or machine form, which are executed by a host during the last part of the file either the final merging instructions have been executed but prior to unloading the output record.

MAGNETIC TAPE CONTROL - A host program that utilizes magnetic tapes for auxiliary storage during a sort.

KEY KEY - The most significant key in a record.

MERGE - A program that performs merging. M-number of merge.

MERGE - The forming of a single file of sequenced records from two or more files of unsequenced records.

SEQUENTIAL SORTING - The systematic sequencing of more than one file, based upon separate parameters for each file, without operator intervention.

INTERNAL SORT - A sort program which is designed to sort data that can be contained within the internal memory of a central computer. Intermediate storage, such as time, tape, drum, etc., is required.

INTERNAL SORTING - The systematic sequencing of a file having more than one input key, without operator intervention.

N - Number of records to be processed by the card. N-1 = maximum number of a type of records. None is capable of processing.

ORDER OF RECORD PROCESSING - The determination of the sequence in which specific records appear in a file should be processed so as to minimize the total volume of merge tapes required to create a single file of sequenced records.

ORDER OF MERGE - The order of input files to a merge program. Also: part of the merge.

GLOSSARY (continued)

FILE - Tapes containing a file in specified sequence as a result of a specific sort/merge process.

FORTRAN - Special coding, provided by the programmer, which is integrated with source/merge coding.

HEADER - One or more records, consisting only of arbitrary characters added to the file to fill out a block (HI - high padding, LO - low padding).

INPUT - The response to the requirement for specifications for a sort/merge operation. Descriptions are used to fix input and output formats, computer configurations, location of keys, and so on.

KEY - The processing of each file record into the purpose of ordering the order of strings of sequenced records and increasing the number of records per string.

LIBRARY - An arbitrary sequence of a sort program. Many sorts are segregated into three phases: initialization phase, internal phase, and merge phase.

LOGICAL RECORD - A technique used in a sort program to merge strings of sequenced data. Since T work tapes, the logic is performed at the power of T-1. The effective power of the merge relation is  $T-1$  and  $T/2$ , depending on the amount of input data and the number of strings.

NUMBER OF LOGICAL RECORDS - Also, way of the merge: order of the merge; the number of input to a merge program. Or, relative order of the merge.

OPERATING SYSTEM - A process also. Also, computer limited. A sort program in which the execution time of the internal instructions determines the elapsed time required to sort. Or, tape limited.

OUTPUT - Data: digital recording.

RECORD - The basic element of a file and the unit of file construction. The number of records of a file is also called the "length".

RECORD LENGTH - A technique used in the internal portion of a sort program. The details of the merge are between groups of records in which the first record is selected between two records with an equal key value. A selected record is then merged with a new record to obtain the selected record. In a merge, a record is selected with the highest key value and the length of the record is compared with the length of the next record.

GLOSSARY (continued)

- RECORD POINT - A point at which a computer program can be restarted in case of computer failure or other reason for inability to continue, without loss of processing accomplished prior to the interruption. Cf: checkpoint.
- RETURN - The return to a previous point in the program where processing may begin. The previous point may be the beginning of the program or it may be a checkpoint. Cf: checkpoint; return point.
- RESTART POINT - See: checkpoint.
- REWARD TIME - elapsed time consumed by a sort/merge program for restoring intermediate and final tape files to original position.
- SEARCH TABLE - See: work tapes.
- SEQUENCE - key number of records (or keys) in the desired sort order.
- SEQUENCE MARK, MARK POINT - That point in a file between the end of one string and start of another.
- SEQUENCING TABLE - See: key.
- INSERT - A method of internal sorting where records are moved to permit the insertion of new ones; also called insertion method.
- SORT - the copying of a file of records into a corresponding file in a specified sequence.
- STAGE, STAGIONAL - The second phase of a multipass sort program, where in strings of data are continually merged until one string of sequenced data is formed.
- SCAN, INTERNAL - The sequencing of two or more records within the central computer memory; the first phase of a multipass sort program.
- SOFT GENERATOR - A program which generates a sort program for production running.
- STRUNG - A group of sequenced records, normally stored in auxiliary computer storage; i.e., disc, tape, or drum.

GLOSSARY (continued)

(N-1)-WAY MERGING - Sometimes called (N). A technique used in a sort program to merge strings of sequenced data. For tape systems that permit backward reading, the effective power of the merge is equal to  $N-1$ .

(N/2)-WAY MERGING - A technique used in a sort program to merge strings of sequenced data. The power of the merge is equal to  $N/2$ .

TAG - The address of a record on disc, drum, or tape, or memory. Sometimes the control field is included as part of a tag record.

TASK LIMITED - Also: I/O limited. A sort program in which the effective transfer rate of tape units determines the elapsed time required to sort. Cf: process limited.

REPLACE WITH SORTING - See: replacement-selection technique.

REPLACEMENT SORTING - See: replacement-selection technique.

VARIABLE-LENGTH RECORDS - Inconvertible file elements for which the number of words, characters, bits, fields, etc., is not constant.

VON NEUMANN PIPE - See: (N/2)-way merging.

WAY OF THE MERGE - See: power of the merge.

WORK TAPES - Also: scratch tapes. Tapes used to store intermediate data when during a sort program.

WAS WITH SORTING - See: replacement-selection technique.



Section VI

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